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METHODOLOGY OF BACK ANALYSIS OF COHESION AND FRICTION OF JOINTS BASED ON PIT SLOPE FAILURE EVENTS

In hard rocks, stability of exposed pit slopes is primarily defined by weak zones in the rock mass, such as joints, contacts of layers, weak interlayers and tectonic faults. The weakening impact of all types of the weak zones is defined by their orientation in relation to one another and to the exposed surfaces, their size, frequency and strength properties which define shear strength along the weak zones. Traditionally, laboratory shear tests define cohesion and friction angle for joints over a small area (approximately 20-30 cm^2). With such scale, it is impossible to model all features of the large joints, such as roughness, undulation and stepping properties. However, it is the large joints of hundreds and thousands square meters in size, along which large failures occur, thus threatening production safety of open pit operations (Figure 1).

Figure 1. Flat (left) and wedge (right) failures of rock mass blocks along joints on open pit slopes

The slope stability guidelines [1] recognise the back analysis based on the actual natural or induced failure events as the most reliable method of shear strength determination. This approach is based on the following assumption: if movement (failure, deformation) of the pit slope have occurred, the shearing forces along the sliding (movement, failure) surface have become equal to the retaining forces. This approach can be fully applied to analysis of various failures which occur from bench surfaces along joints. The back analysis allows obtaining practically valuable geotechnical information from the actual failure events, specifically the data on cohesion and friction angles along the joints.

The back analysis of shear strength along the joints represents a passive experiment undertaken by the nature. In the process of mining, we only create conditions for shearing along the joints of rock mass blocks by forming benches and berms. The shear test itself is then performed by the forces of nature. The passive full scale experiment has the following significant differences from laboratory scale rock testing:

- The sizes of failures on the pit slopes exceed the sizes of laboratory tested samples by hundreds and thousands times. Therefore, the function of scale is taken into account;
- In full scale conditions, a bench, which is a constructive element of the pit slope with real geological structure, is tested by a combination of natural (gravity, own weight of the rocks) and

induced (seismic impact of blasting operations) forces which actually exist in the pit. In laboratory conditions, it is impossible to model the real conditions of stress on the open pit benches. Therefore, the passive experiment takes into account all factors which impact the actual pit walls;

- Benches in the pit stay under stress for years and decades; laboratory testing takes minutes. Therefore, the passive experiment takes into account the time factor, which includes such additional factors as seasonal freezing and thawing processes, periodical flooding and permanent deterioration of rock mass as a result of regular seismic impacts of blasting.

The above listed differences make the full-scale data obtained by back-analysis significantly more valuable and substantial than the data of laboratory tests. This study presents a methodology for back-analysis of cohesion and friction angle along joints based on actual flat or wedge failures on the open pit benches.

When using the Mohr–Coulomb criterion in the back analysis, the shearing resistance of exposed rock blocks along their delineating joints consists of two components - the forces of cohesion and friction. The source of shearing force is the own weight of the unstable rock mass block. Shearing (failure, deformation) of rock blocks from the benches occurs where and when the shearing force reaches the level of shear resistance. The principle of back analysis consists of an analysis of a failure event under condition of equal shearing and resisting forces, i.e. in a limit state with factor of stability equal to 1. Cohesion and friction angle along joints are back calculated according to the following algorithm.

For each failure, cohesion (C') and friction angle (φ') are back-calculated along the joints where the shearing occurred using the field measurements of the failure geometry (failure dimensions, dip angles and dip directions of the relevant joints). Because it is impossible to calculate two unknown values (C' and φ') from a single failure event, the back-calculation for each failure is performed on an option by option basis, with defining one unknown value (for example, the friction angle) and back-calculating the second value (cohesion) based on the Factor of Safety = 1 for the sheared block. The very convenient back analysis tools are Swedge (Rocscience Inc., Canada) and Sblock (E. Esterhuizen, SRK Consulting, 2004). Figure 2 shows the screenshots of Swedge software.

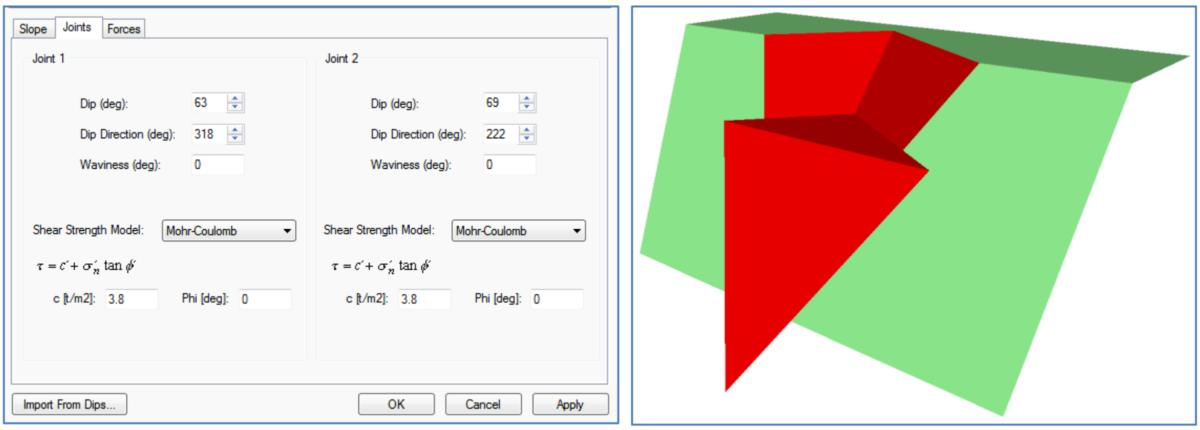

Figure 2. Menu for definition of joint geometry (Dip, Dip Direction), shear resistance parameters along the joints (Shear Strength Model: C' and φ' - cohesion and friction angle) and type of modelled wedge failure from the bench

As a result of the option by option calculations, curves are drawn for such combinations of cohesion and friction which ensure the limit equilibrium of the failure along the logged joints. By the set of failures in the coordinates of C' and φ' it is possible to obtain a number of curves.

Each curve describes a set of possible joint strength properties in each failure. The expected values of cohesion and friction for the set of failures can be found by intersection points of the curves. Figure 3a shows an example of such analysis based on three failures at one of the open pits in Chile. The intersection points define that the most probable cohesion value is in the interval of C' = 7.0-7.5 kPa, and the friction angle value is in the interval of φ' = 26-31°. Figure 3b shows another example of back analysis of 7 options of a large wedge failure measurements on the north-eastern wall of North-Vorontsovsky open pit. In this case, all seven lines intersected at one point with the coordinates of C' = 0 and φ' = 26.4°. This is a rare case of ideal convergence of the full-scale data which suggests that the strength properties have relatively small variability.

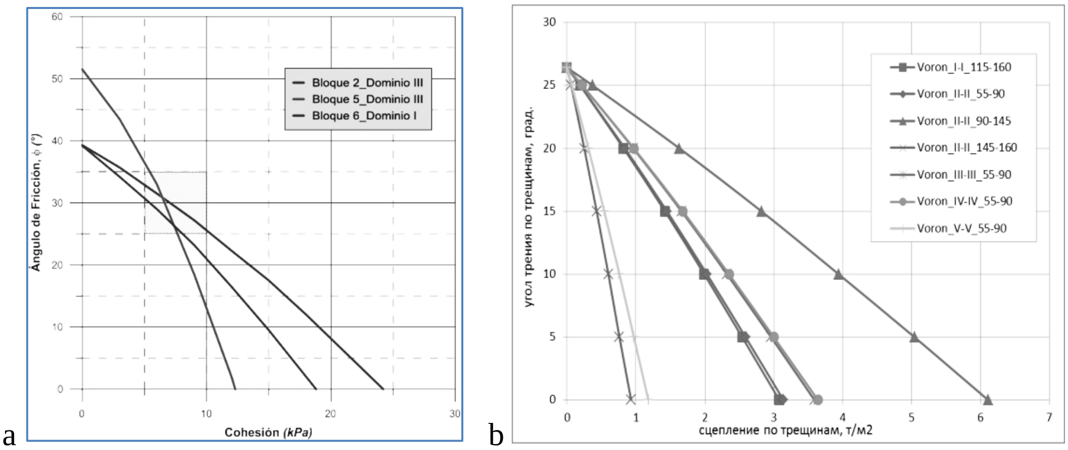

Figure 3. Back analysis of cohesion and friction angle along joints by a set of failures

With a large amount of back analysis data on a large set of failures, the intersection points $\varphi' = f(C')$ can be found if the back-calculated curves are approximated by regressions. Special software is available to do this. Figure 4 shows the back analysis results for 32 wedge failures with a volume from few cubic meters to several thousand cubic meters from the benches of Eastern Pit at Olimpiada Mine. Field mapping of the failures was performed by geotechnical laboratory of Polyus Gold. Using the software, 308 intersection points of 32 curves were found.

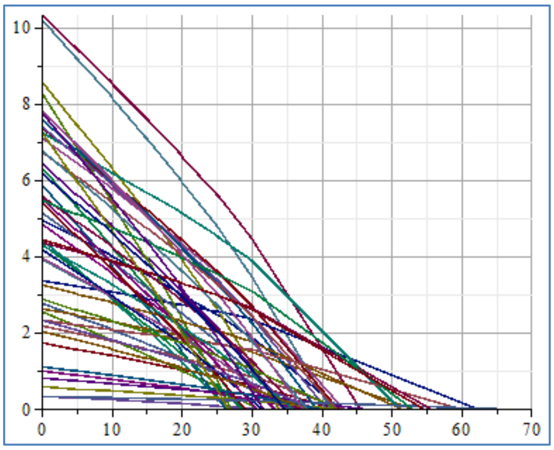

Figure 4. Results of back analysis of cohesion and friction angle based on wedge failure events at the Eastern (Vostochny) pit of Olimpiada Mine.

The resulting data were subjected to statistical analysis. First of all, the unreliable (significantly different) data were rejected using the Smirnov-Grabbs criterion. After the initial data set was reduced, the distribution laws for shear resistance parameters were found for the remaining selection of 287 values.

Distribution of cohesion along joints (C') follows the negative exponential law (Figure 5a):

$$p(C') = \lambda \cdot exp(-\lambda \cdot C')$$

where λ is the parameter of exponential distribution equal to 2.30 for joints at Olimpiada. The exponential distribution law is a one-parametric law, in which the $1/\lambda$ value is both the average value and the standard deviation of a random value. Therefore, according to statistical data, the average cohesion along joints is $C' = 1 / 2.30 = 0.43$ t/m^2 = 4.3 kPa. The exponential distribution law means that the loss of cohesion force during shearing along the joints develops like an avalanche, according to the weakest link concept. At the limit, the mode (the most probable value) of the parameter distributed according to exponential law tends to zero.

The distribution of friction angle along the joints (φ') follows the normal law (Figure 5b) with average value of 30.6° and the standard deviation of 7.3° (variation factor of 24%). Physical basis of the normal distribution law consists of the process of friction force reduction during shearing along joints in accordance with classical beam theory, i.e. it is defined by a large amount of random events with any kind of distribution. Figure 6 contains a bi-variant histogram showing the exponential distribution of cohesion (C') and normal distribution of friction angle (φ').

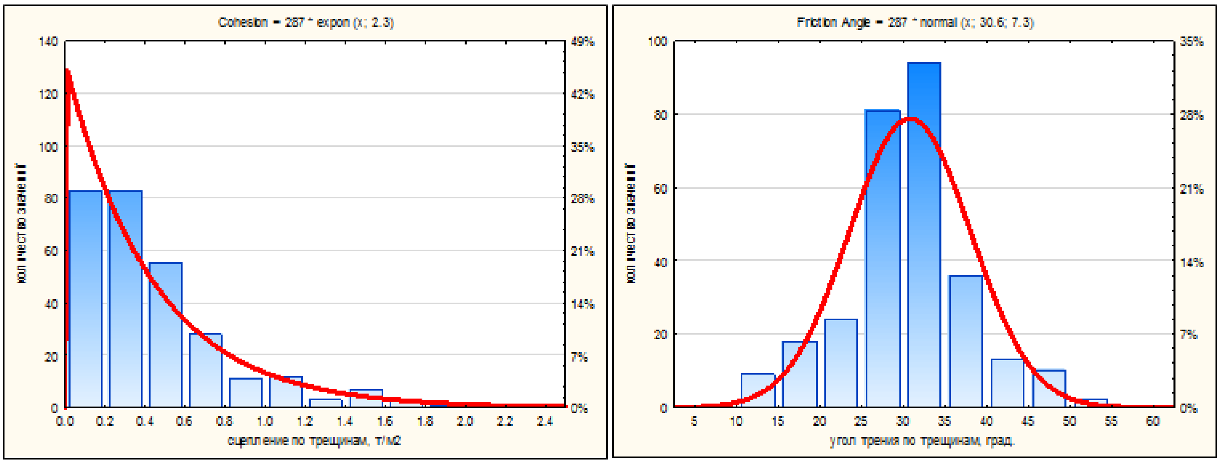

Figure 5. Distribution of cohesion and friction angle along joints for the whole set of analysed data

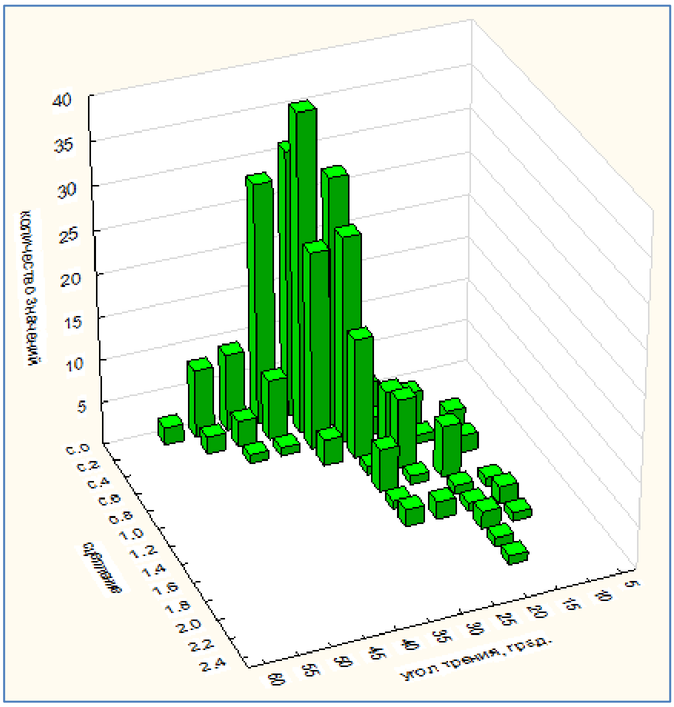

Figure 6. 2D distribution of cohesion and friction angle along joints in the overall set of back-calculated data

To reduce the variability of joint strength parameters, the general set of data can be divided into groups by some geological properties (lithological type, bedding depth taking into account weathering, degree of dynamic metamorphism). The attempt to divide the general set of data into three selections by the key lithological types was unsuccessful: the differences of shearing resistances along joints between different types of metamorphosed rocks at Olimpiada were within their statistical variation.

To estimate the factor of scale, the back analysis results for shearing resistance parameters were compared against the shear testing results obtained by the geotechnical laboratory of Polyus Gold (see table below).

Comparison of laboratory testwork results versus back analysis results for shear resistance parameters at Olimpiada Project

	Laboratory testwork	**Back analysis**	**Laboratory testwork**	**Back analysis**
	friction angle, degrees		***cohesion, t/m^2***	
Distribution law	normal	normal	lognormal	exponential
Average value	***30.2***	***30.6***	***5.70***	***0.43***
Standard deviation	8.5	7.3	5.16	0.43
Variation factor	28%	24%	90%	100%
Amount of values	88	287	85	287

The comparison showed that the friction angle values were the same in the laboratory scale and in the full scale conditions. The difference in the mean values was statistically insignificant. This allows combining the selections of laboratory and full scale results into one general set of 375 values with average value of 30.6° and mean square deviation of 7.6° (variation factor of 25%). The similar average, median (30.8°) and mode (30.0°) values confirm that friction angle distribution by joints follows the normal law. Physically, this means that the laboratory scale of friction measurement over the area of 20-30 cm^2 is quite enough to conform to the central limit theorem by A.M. Lyapunov and can be applied to the full scale conditions without any amendments. This conclusion obtained for metamorphosed rocks of Olimpiada deposit (carbonate-quartz rocks, carboniferous cataclasite, shale and dynamic shale, quartz-mica shale) was unexpected to the authors of this study. It was expected that the friction angle along joints in full scale conditions would be greater than that in the laboratory conditions due to longer joints, their roughness and undulation. The difference in the angles was expected to be equal to undulation angle. Possibly, the fact that the friction angles are equal is relevant only for the metamorphosed rocks with shale structure and small (within statistical variation) undulation angles.

The situation with cohesion is different. According to the back analysis results, cohesion along joints in full scale conditions is smaller than the cohesion in laboratory conditions by an order of magnitude (by 13 times). It is notable, that the distribution law also changes: in the laboratory scale, the cohesion values along joints are distributed according to lognormal law, and in the back analysis data the values are distributed according to exponential law. This means that the increase in testing scale from laboratory to full scale (i.e. increase by hundreds and thousands of square meters) causes a scale effect of cohesion reduction, with cohesion value tending to zero.

In international practice of bench-berm design [2] it is recommended to accept cohesion along joints as zero, i.e. to apply conservative approach with lowest (zero) cohesion value distributed in accordance with exponential law. As a rule, such approach is applied at the mine design stage (green field stage). At the stage of operations, when practical data of rock mass behaviour are available along with the opportunity for back analysis of actual failure events, designers (consultants) can select the real value of cohesion along joints. This more optimistic approach is designed to achieving more aggressive and therefore more risky, but also more economically attractive pit slope parameters.

CONCLUSION

1. The proposed methodology of back analysis of cohesion and friction along joints based on the option-by-option analysis of a set of failures was tested at several open pits and proved to be applicable.

2. According to laboratory testwork and back analysis results, the distribution of cohesion along joints follows exponential law, and the distribution of friction angle values follows normal law, which is related to various mechanisms of shearing resistance reduction in the cohesion and friction forces.

3. In the metamorphosed rocks with shale structure at Olimpiada deposit, the difference between the laboratory and full scale results for friction angle along joints was statistically insignificant, i.e. the friction forces were not affected by the scale of the process, because the undulation angles of the joints were within the statistical variation range of friction angles along these joints.

4. According to the back analysis results, cohesion along joints in the full scale conditions was smaller than the cohesion in laboratory conditions by an order of magnitude. This reflects the sensitivity of cohesion forces distributed according to exponential law to the scale of

the process: the greater the joint area, the more likely that the joint will contain a section (fragment, element) with zero or very low cohesion which initiates the shearing process.

5. Design of bench-berm parameters can be based on the friction angles along joints measured in laboratory conditions. Selection of the cohesion value along joints depends on the project strategy (conservative/pessimistic or aggressive/optimistic) and the acceptable level of risk. The conservative option is the most preferred at the initial project stages, when the lowest (zero) cohesion along joints can be accepted. At an operation stage, if the design solutions are revised (for example, for a pit depth increase) and the project economic model needs to be optimized, an optimistic option can be implemented. In this case, the cohesion value can be estimated by back analysis based on the actual failure events.

References:

1. *Slope stability guidelines for coal open pits. - VNIMI, 1999.*
2. *Guidelines for open pit slope design / Editors John Read, Peter Stacey. - CRC Press/Balkema, 2009. – 509 p.*